Soap films: Nature behind the Maths

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June 17, 2011

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*This is a final project for the course Historical Aspects of Classroom Mathematics conducted by Prof. Steven Wepster at Utrecht University, Spring 2011. The authors wish to acknowledge colleagues Nick Evans and Olaf Hammelburg for their inestimable peer review.
Abstract

In this paper we explore the links between nature and mathematics through the example of soap films and bubbles, in particular their potential to help find a solution to some difficult optimization problems. The mathematics behind soap films are concisely exposed, along with some physical indications. As a ready-to-use application we extensively develop a teacher’s handbook on soap films: how soap films adopt surfaces of minimal energy when constrained to whatsoever bounds, together with instructions on how to construct such bounds with wire, how to prepare the soap-and-water solution and increasingly difficult questions to be posed to the target group.

This paper is aimed at high school mathematics teachers with pupils ranging 14-16 years old.

1 The nature of maths

What do the Kepler conjecture, the Pigeonhole principle and Plateau’s laws for minimal surfaces have in common, aside from being important results on high-level mathematics? Remarkably, all of them can be ‘proved’ using objects within reach of children. Indeed, every fruit seller knows that oranges in an alternated pile take up the least space, and that seven pigeons in six pigeonholes forces one to share room\(^1\). If the reader deals with motivating students toward mathematics in her everyday life (some prefer to call them ‘maths teachers’) then a switch may have clicked in her head: nature as a hook!

Nature is par excellence the source of inspiration for scientists. Clearly, mathematics were born this way: early mathematical objects reflected —or more precisely, were— what can naturally be found in the surrounding world. Arguably, formalism may have come with Euclid’s axioms—nevertheless a ‘point’ or a ‘line’ remained to be undefined notions that were taken for granted. But essentially most simple mathematical concepts have a counterpart in nature.

Symbols and equations have helped mankind deal with practical problems, rigorously proving the obvious and not-so-obvious and providing with unexpected solutions that may have remained unnoticed otherwise\(^2\). In this paper we show that an inverse method can sometimes be not only possible but also didactical and enlightening, exemplified in the solving of the problem of surface optimization with the help of soap films. The beauty of the

\(^1\)So what about minimal surfaces?
\(^2\)Take the two imaginary solutions of \(x^3 - 1 = 0\) as a childish example.
method also deserves some attention, since motivation plays a key role in learning.

1.1 Relation between the project and the ICMI framework

ICMI stands for International Council for the Teaching of Mathematics, an organization who makes suggestions on how to organize history in education and especially the history of mathematics in the classroom. In this section we discuss the suggestions of the ICMI with references to the numbered themes in the ICMI-study [12].

In this project a brief history of the study of minimal surfaces is presented. Information is added about the various important scientists who encountered the problem of minimal surfaces and proved some important properties of soap films and soap bubbles (ICMI 7.3.1).

At a few moments in this project the students, our target group, will carry out the same experiments as one of the scientists did 150 years ago. The students will then analyse their experimental results and come to conclusions just the way this scientist did.

In our handbook there are exercises with worksheets to be done, conceived to develop the scientific mind of the students. There they discuss results with each other. The worksheets are aimed for them to grasp the subject and its properties. It will hopefully lead the students to the development of the basics of a previously unknown topic (ICMI 7.4.4).

The subject also belongs to problems presented for recreational purposes (ICMI 7.4.7 (v)). Which soap films or soap bubbles exist with different wire frames? In addition to this there are mathematical concepts and proofs using instruments, like different two- and three-dimensional wire constructions to put soap liquids in. The students will play with copies of instruments, which scientists of earlier times used, to see what kind of shapes they will produce (ICMI 7.4.8). These instruments will somehow “prove” the mathematical results for minimal surfaces and provide with solutions to other mathematical problems.
2 The maths behind the soap

The aim of this section is to provide the high school teacher with compiled material enough to understand the behaviour of soap films in a considerable depth. This material may be expanded with other resources that the teacher deems appropriate—for this the bibliography might be helpful. Full comprehension of the section qualifies the teacher to proceed with the practice, for a ready-to-use handbook is presented in Section 3. It features exercises of increasing difficulty and thorough instructions on how to set up the soap and the wireframes, all arranged for two lessons of 45 minutes. The recommended target group is VWO/Havo 4 (14 to 16 years old)\(^3\), though it may be easily adapted to other groups.

2.1 A bite of history

The synthetic study of liquid surfaces, which has led to our nowadays knowledge of soap films and soap bubbles is thought to date from the time of Leonardo Da Vinci. He studied the rise of a liquid up a capillary tube when it is inserted in a bath of liquid.

Since the fifteenth century researchers have carried out investigations on liquids through two distinct approaches. In one approach are the physical, biological and chemical scientists who have studied the macroscopic and molecular surfaces of liquids. In the other approach mathematicians have been concerned with problems that require minimization of the surface area contained by a fixed boundary. A simple example is the minimal area surface enclosed by a wire, which is created after submerging the wire into a soapy liquid. \cite[8, p.5, p.40]{8}

The fantastic forms of soap films and their mathematical models are visual and very striking examples of minimal principles. Mathematicians through the ages have been inspired by them not only for their beauty but also because the subject contains interesting and challenging mathematics.

Bernoulli (18\(^{th}\) century) applied methods of calculus to the solution of minimal surfaces. Lagrange, in turn, was able to write down mathematical equations for the determination of minimal area surfaces (1760). However, the most extended experimental and theoretic work on the field of minimal area surfaces was done by Belgian physicist Joseph Plateau (1801-1883). He carried out extensive and fascinating experiments with minimal surfaces, soap films and membranes. In 1873 he published most of his conclusions in the \textit{Statique expérimenterale et théorique des liquides soumis aux seules forces}
moléculaires. [3, p.241]

All minimum surfaces were found to have some common geometric properties, which the experiments of Plateau showed. Plateau realized that every contour of one single closed wire encloses at least one soap film. Is there a similar result for the mathematical models of soap films, namely, minimal surfaces? This is one of the problems of Plateau: given an enclosure, to prove that there is a minimal surface within that enclosure. He studied two- and three-dimensional figures and this led him to the now called three laws of Plateau.

Plateau became blind by a dangerous experiment. He looked into the sun for more than 25 seconds and damaged his eyes so. Although handicapped he continued his studies with the help of his family and friends.

Plateau’s achievements inspired and motivated mathematicians to re-encounter the problems of minimal area surfaces. It attracted the interest of mathematicians and has led to fruitful interaction between the earlier mentioned two approaches. Lots of mathematicians tried to solve the problems of Plateau, but it was not until 1931 when Jesse Douglas came up with a solution of this and other problems on the area of minimal surfaces. Douglas got the highest award in mathematics (Fields medal) in 1936 for all his work. Indeed he proved that for every single closed curve a surface with minimal proportions exists that tenses that curve, and this minimal surface has no cuts to itself.

The three laws of Plateau were mathematically proven by Jean Taylor in 1976. [11]

2.2 Modelling soap

Soap films are fragile structures roughly composed of a thin layer of water confined amongst two layers of soap molecules. The particular structure of soap prevents water from spreading out, the hydrophilic head of the molecule against the water and the hydrophobic tail towards the air, as shown in Fig. 2. Like many other structures in nature soap films tend to minimize energy. This change of shape naturally occurs in a continuous way, thus the stable shape eventually adopted will always be that of local minimal energy, but not necessarily global. This suggests that more than one shape may be
Figure 2: Microscopical structure of a soap film. Water is sandwiched between hidrophile heads and hidrophobe tails of soap molecules.

possible for a given boundary.

Consider a soap-and-water solution blown into the air, i.e. with no external constraints. The shape adopted, every kid knows, is that of a sphere. The reason for this is that the sphere confines a given volume in the smallest area, the volume being the only constraint. Let us now constrain the soap to a circumference, for example one made with wire. The soap film obtained will be flat, filling the interior of the circumference like a drum head, since any different shape would encompass a greater area. The same principle applies with more complicated boundaries. If the wire is bent as in Fig. 3 the

Figure 3: Bending of a hoop holding a soap film. Note the lifting of the film in the middle part.

resulting soap film will no longer be flat, but still of minimal area.

New situations arise when two films coincide. Surprisingly, they meet in three by three in a common straight line, forming dihedral angles. If these lines coincide, they do in four by four forming trihedral angles.

After intense field research, Joseph Plateau recorded these rules as those governing soap films:

**Theorem** (Plateau’s laws).

1. *Three smooth surfaces of soap film intersect along* a line.

2. *The angle between any two tangent planes to the intersecting surfaces, at any point along the line of intersection of three surfaces, is* $120^\circ$.

---

For practical reasons air resistance and gravity will be discarded.
3. Four of the lines, each formed by the intersection of three surfaces, meet at a point, and the angle between any pair of adjacent lines is \(109^\circ 28'16''\) \((\approx 109.47^\circ)\).

Where do these numbers come from? What about that fanciful 109.47? Notice that \(\arccos(-1/2) = 120^\circ\) and \(\arccos(-1/3) \approx 109.47^\circ\), suggesting a simple origin. Deduction goes as follows.

Assume Plateau’s laws to be true except for the specific angles, namely, (1) three smooth surfaces of soap film intersect along a line, (2) the surfaces forming equal angles and (3) the lines meeting in four by four at a point with equal angles. In property (2) little explanation is needed to show that the angle must be \(360^\circ/3 = 120^\circ\) (see Fig. 4).

![Figure 4: Angle formed by three surfaces of soap film.](image)

For property (3) consider a tetrahedron with vertices at \(A = (0,0,0)\), \(B = (0,2,2)\), \(C = (2,0,2)\) and \(D = (2,2,0)\), as shown in Fig. 5. From each of these vertices we throw a line to a common point \(P\) with the condition that \(P\) is equidistant to \(A, B, C\) and \(D\). Clearly \(P = (1,1,1)\). Now, by symmetry, the angles between the four lines meeting at \(P\) are equal, so they must be the angle we are after. How to compute it?

Let the angle to be measured be \(\alpha := \angle APD\). Thus we will take interest in vectors \(\overrightarrow{AP} = (1,1,1)\) and \(\overrightarrow{PD} = (1,1,-1)\). Two ways of computing the scalar product lead us to

\[
\overrightarrow{AP} \cdot \overrightarrow{PD} = 1 \cdot 1 + 1 \cdot 1 + 1 \cdot (-1) = 1
\]

\[
= \frac{|\overrightarrow{AP}| |\overrightarrow{PD}| \cos(\pi - \alpha) = 3 \cos(\pi - \alpha) = -3 \cos \alpha}
\]

so \(\alpha = \arccos(-1/3) \approx 109.47^\circ\), as we wanted to show.

It is worth pointing out that Plateau deduced these laws by mere observation. A rigorous mathematical proof was not found until 1976 by American mathematician Jean E. Taylor [11].

\[5\text{Beware, should the figure not be a tetrahedron, this } P \text{ could not exist!}\]
2.3 Minimal surfaces

In differential geometry a minimal surface is that with mean curvature zero. What is the mean curvature? In every point of a “smooth” surface there are two principal curvatures, denoted as $k_1$, $k_2$, which measure the smallest and greatest bending of the surface in that very point. Moreover, the directions of these two bendings are perpendicular. The mean curvature is defined as $H := (k_1 + k_2)/2$, so when both curvatures oppose one each other with the same intensity, i.e. $k_1 = -k_2$, then $H = 0$. Every surface that satisfies this in every point is called a minimal surface. Suppose that the surface is a soap film characterized by having a surface tension, this is, the elasticity forcing the film to adopt the surface of minimal area. If both principal curvatures “pushed” in the same direction, i.e. $k_1 k_2 \geq 0$, then the surface would diminish its area by moving towards that direction. Since this cannot happen in a soap film, both curvatures must not only push in opposite directions but in fact cancel out, hence $H = 0$.

For a long time the only infinitely-extending surfaces without self-intersections known to be minimal were the plane, the catenoid and the helicoid. In the last decades numerous examples of other such minimal surfaces have been found [1]. When a boundary is required and self-intersections are allowed, soap films arise as a powerful tool to find new ones, as it is the case of the canopies of the Olympiastadion in München (Fig. 8).

For a rigorous exposition of the subject, Do Carmo [6] is a must.
2.4 Concerning shapes

We have seen that a soap film changes its shape continuously until it reaches a shape of local minimal energy. To illustrate this consider the two possible configurations shown in Fig. 9. How can the catenoid possibly exist, the two discs having a smaller area? Well, both are local minimal surfaces with energy $E_1$ and $E_2$, respectively, as in Fig. 10. If we blow both discs together they will eventually merge into one and adopt the shape of the catenoid. Energy has been transferred into the system, violating the minimal energy property and allowing the surface to overcome the pit of $E_1$ and falling into the pit of $E_2$.

In general, a wireframe may allow many minimal surfaces, each corre-
sponding to a local minimum of the energy function. Blowing, pricking or other external forces are the key to jump from one another.

2.5 An application: Optimization in 2D

When a soap film is confined to a narrow space, for instance, between two parallel flat surfaces—from now on referred to as “sandwich”—, a two-dimensional problem can be implemented. It is commonly known as “the motorway problem”:

Given a number of towns, which is the shortest motorway system linking all of them?
Consider the case of four towns arranged in the corners of a square of side 1 km, as in Fig. 11. Four layouts are shown, but there is no clear strategy to find the optimal one. However, theory of soap films confirms that (d) is in fact the shortest path (each angle being 120°). This can be checked with a transparent sandwich-structure joined together by four rods perpendicular to the “slices”, the rods serving as the four towns (Fig. 12). Thanks to the first Plateau rule we can discard depth, for it provides no additional information, thus reducing the three-dimensional problem to two dimensions.

By extending this example to any number and display of towns one should be able to solve every motorway problem encountered. This is a relief, since no general analytic solution exists.

2.6 Pedagogy

Consider starting your high school lecture about soap films with the following result:
The singular set of an \((M, \xi, \delta)\) minimal set consists of Hölder continuously differentiable curves along which three sheets of the surface meet (Hölder continuously) at equal \((120^\circ)\) angles, together with isolated points at which four such curves meet bringing together six sheets of the surface (Hölder continuously) at equal angles. [11, p.489]

Do we expect any reaction other than massive loss of attention? Yet this is the mathematical equivalent to Plateau’s laws proved by Taylor in 1976. Instead, consider the following warming-up:

**Exercise** Dip a wireframe with the shape of a cube into a water-and-soap solution. What shape does the soap adopt? Why?

Even if they do not know the answer, as they will certainly not, that “why” is intentional. Unexpected solutions have a potential to stimulate proper thinking and re-building intuition [4]. Moreover, a number of studies show that practice preceding theory boosts children’s comprehension of mathematical concepts [12]. In this respect the pedagogical method proposed here emphasizes the need of letting the pupils play with soap before going on with the theoretical explanations.

As to the physical and mathematical content put forward earlier, the students are not expected to learn but what belongs to their level, which is little of the aforementioned. However, the teacher must be ready to answer questions arising out of their curiosity, like what is a soap film composed of, why does it search for minimal energy configurations, etc. An enriching debate may come up spontaneously and she must be prepared to deal with it and channel it onto a constructive path. Moreover, it is her duty to stimulate
this debate if the workshop has failed to. We strongly defend this method at this stage of their learning, as opposed to the theorem-proof-example procedure. Whether or not curiosity arises, it should become clear for the students, and so must the teacher transmit it, that difficult mathematics lie behind those wireframes and that, luckily, nature comes to our aid.
3 A workshop on soap films: ‘Nature behind the Maths’

In this part we propose a program for two lessons in classroom aimed at a target group of VWO/Havo 4 in the curriculum of Wiskunde B [2, pp.1–11]. We think that these lessons could be planned parallel to Chapter 2 ‘Oppervlakten en inhouden’ (‘Surfaces and contents’) of the textbook of the curriculum [10].

This part consist of

1. introduction for the teacher,
2. textbook with exercises for the students,
3. worksheets to be needed.

3.1 Introduction for the teacher

How can you find the shortest roadnet between a certain number of locations? How to design the roof of a stadium with the smallest area? These minimization problems are not easy to solve theoretically. The study of soap membranes and bubbles gives a practical solution to them. In this sense, nature comes to the aid of mathematics; here is the central theme of this project. In the student handbook which is presented here, there are exercises for two lessons. For every lesson you will find a small list of needed materials, a short overview of the lesson, a description of the demonstrations and practical experiments. History is incorporated by telling the students some of Plateau’s background and by highlighting that during these lessons they will ‘lie on the shoulders’ of Plateau by simulating his experiments.

The perspex sandwich An essential tool for these lessons is the soap film sandwich (Fig. 13): two perspex plates containing little holes to put nails across. The authors have worked with a sandwich measuring 8 by 12 cm, with 1 cm separation between the plates (Fig. 14). If you dip the sandwich into a bucket of soapy liquid, soap films will link the nails in particular shapes. Extra holes can be bored to create different figures.

The teacher of manual skills at your school may be willing to help you make several sandwiches. For an optimal ‘practice preceding theory’ it is worth having these instruments in a rate of one per two or three students.
The wires of Plateau  Another tool to be needed are the wires of Plateau, or simply wireframes (Fig. 15). They represent closed curves as in the problem of Plateau, which in our case will resemble most familiar shapes like cubes, tetrahedrons or hoops.

Again, the teacher of manual skills can help you make them.

The soapy liquid  Use normal liquid soap for daily use, in which the essential ingredient glycerin is contained. Take a tank with warm water and slowly pour the liquid soap into it. Stir carefully—bubbles must not appear. The ratio between water and soap is ten to one.
3.1.1 Lesson one: Soap films in two dimensions

We strongly advise to read this section with the education textbook for the students next to it. This will help you to understand it more quickly.

Materials

- Geo-triangle (ruler and protractor —angle measurer—)
- Graphic calculator
- Transparent paper (size A4)
- Soap film sandwich with two patterns of holes: one as in Worksheet 1 and one in a square with sides 6 cm
- Soapy liquid
- Worksheet 1

Overview In the exercises students in groups of two or three analyse the problem of finding the shortest motorway between four points. First they play with the sandwich in the liquid and draw the produced soapfilm with paper and pencil. Some of Plateau’s background is given (notice that the three laws of Plateau are not yet mentioned). Tell the story of his fatal accident!
Then the students are encouraged to compute the total length of a minimal roadnet and do some exercises. The possible solution of these exercises is to be found in the amount of degrees of the angles made by the roads. This can be made visible by the soap films in the sandwich, to be dipped by the students into the liquid. Let the students discuss what they see and link this practice with the exercises about the total length and angles made.

About the history you talk and inform the students about the first two laws of Plateau. And most important: they have to understand that one can optimize the length of a roadnet through computations, but nature gives you the answer right-at-hand.

3.1.2 Lesson two: From 2D to 3D

Materials

- Geo-triangle, compass
- Transparent paper
- Ruler
- Drinking straw
- Small glass plates to blow bubbles on
- Wireframes: cube, tetrahedron, pyramid and two rings, all sized approximately $10 \times 10 \times 10$ cm
- Soapy liquid
- Worksheet 2

Overview In the beginning of this lesson the students see how materials behave to get minimal surfaces. Soap bubbles behave the same also when several bubbles lay to each other. The students survey soap bubbles on one flat surface. The angles which are formed between the three similar soap bubbles are again 120 degrees. Then the students look at the soapfilms in three-dimensional wires, which they put themselves in the soapy liquid.

In this section no mathematical work is asked, but only questions that refer to previously acquired knowledge of two-dimensional soap films. The teacher is now able to explain all Plateau’s rules (including the third law) and some further theoretical results that the students will easily grasp. We assume the third law to be too difficult for the considered target group. Hence
we advise only to tell the content of the law itself, for which Fig. 5 might be illustrative.

The teacher may want to perform a practical demonstration of soap-film blowing—see students practicum 1 and 2. If time allows, it is advisable to spend a few minutes letting the students try by themselves before going through the exercises.

**Students practicum 1: to blow bubbles** The blowing of bubbles is carried out with a straw submerged into the soapy liquid and blown onto the transparent paper or on the small glass plate. Blow until you get a half sphere. This can be measured with a ruler, the height equal to half the diameter. Measuring is also possible by putting the (wet) ruler through the soap bubble.

To blow two bubbles: It is important to blow bubbles of a similar shape. The student can blow twice with one straw at a time or once with two straws at the same time. Then a flat surface will be formed in between.

**Students practicum 2: to blow soap films** Use wires of Plateau with the shapes of a cube, a tetrahedron and a pyramid, to be put through the soapy liquid. Extract the wireframe carefully. Then shake it gently to show how the soap film returns to its initial shape. If two different stable shapes are obtained for the same wireframe, postpone the explanation until the end of the workshop.

### 3.2 Education textbook

**3.2.1 Lesson one: Soap films in two dimensions**

**Worksheet 1** After a short introduction by the teacher you put six nails across the sandwich in the way shown in Worksheet 1. Now dip the sandwich into a bucket of soapy liquid. A soapfilm will appear linking the nails. Try to draw this figure as seen from above (top view). What you have just done was what a famous scientist, the Belgian Joseph Plateau, did 150 years ago. Listen to a short story by your teacher about this scientist and other developments in the past centuries.

Now you go on with a mathematical problem: how to optimize the length of a roadnet connecting four points?

1. Draw the shortest route you can think of between four points on a square with side 6 cm and measure the length. See Fig. 16.

2. Do it again but try it to be shorter.
3. Try to look at curves instead of straight lines. Do they give a better answer?

Look at Fig. 17 and draw it in your notebook. Call the left midpoint $A$ and the right midpoint $B$. Try to think that all the links, also between $A$ and $B$, are rubber bands. Now you start to analyse the total length of the roadnet when you put the bands to each other.

![Figure 17: A and B are respectively the midpoints of the left and right roads. Think of the roads as rubber bands.](image)

4. (a) Draw in a new graph a roadnet where $A$ and $B$ lie closer to each other than in Fig. 17. Is the total length greater or smaller?

(b) What will happen with the total length?

(c) What happens when $AB = 0$?

5. Imagine that the figure is not symmetric but that $B = A$. What you see is now a K-figure instead of an H-figure. Which figure gives the shortest length?
6. (a) Take for \( \overline{AB} \) respectively the amounts 100, 80, 60, 40, 20 and 0 meters and calculate the length of the roadnet. Let the figure be still symmetric. Write the results in a clarifying table.

<table>
<thead>
<tr>
<th>Length ( \overline{AB} ) (m)</th>
<th>100</th>
<th>80</th>
<th>60</th>
<th>40</th>
<th>20</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total length (m)</td>
<td>300</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(b) What is your conclusion now?

7. (a) With which amount of \( \overline{AB} \) is the total length of the roadnet minimal?

(b) What is that length?

8. (a) Draw the best solution you found in Exercise 7. Write down the length of each piece in your drawing.

(b) At \( A \) three roads come together. Measure how big the angles are.

(c) If you look at the lengths you have written down, you can compute with these lengths the angles at \( A \). With what rules of the mathematics can you compute them? Write down the computations in your notebook.

You have just obtained with some computations the minimal length of a roadnet in a situation of four points, which are the points of a square. What is most important is the finding of Exercise 8(b): the measurement of the angles at \( A \).

9. Take the sandwich again and put the nails in it, such that they make a square with side 6 cm. Dip it in the liquid. Draw the figure you see as accurately as possible on a transparent paper. Put the transparent paper over your solution of Exercise 8(a). Discuss what you see.

10. Take your drawing of Exercise 1 and measure with your geo-triangle the angles made between the soapfilms. Discuss and try to extract similarities between all the drawings.

You have now carried out some of the experiments of Plateau and luckily have come to similar conclusions: the first two laws of Plateau. Your teacher will conclude this lesson with a short summary about these two laws.

3.2.2 Lesson two: From 2D to 3D

In lesson one you have learnt about the two laws of Plateau and you ‘lay on the shoulders’ of Plateau by replicating his experiments. Remember one of the laws: the angles on which soap films meet are of 120 degrees. In this lesson we look further to three-dimensional figures.
Worksheet 2

11. (a) Start by blowing bubbles with a straw and the soapy liquid. Try to produce two bubbles of equal size and let them ‘meet’.
(b) On Worksheet 2 you see four drawings of the top views of soap bubbles. Try to blow these figures on a piece of glass plate with a straw.

12. Soap bubbles join each other always on a certain manner, because of the tension in the soap film.

(a) Which drawings of Worksheet 2 do not work? How did you see that? Why do the other drawings indeed work?
(b) See figure B of Worksheet 2. Think of a way to find very precisely the midpoint of each circle and draw that midpoint. Is it indeed so that there is again an angle of 120 degrees? Where can you find it?

13. Draw on a transparent paper a top view figure of two soap bubbles, each with a radius of 4 cm. Use what you already know and check this drawing with the blowing on the transparent paper of two equal soap bubbles of 4 cm radius each.

Back to Plateau: he also studied 3-dimensional figures and their characteristics.

14. Take the tetrahedron and the cube wire frames.

(a) Discuss what will happen when they come out of the soapy liquid.
(b) Get the wire through the soapy liquid and discuss what you see.
(c) Draw a front and a side view of the wires with their soapfilms.
(d) What kind of angles made by the soap films do you measure in your drawing? Again 120 degrees or any other ones?
(e) Use a pyramid and two rings wire frames to find out what properties hold in all of them (universal properties).

You have reached the third Plateau’s law.

On our journey through soap films we have extracted three universal laws. The reason for them even Plateau could not deduce (the answer was not found until 1976!). What Plateau did know was a physical property that he guessed was behind them: soap films always look for shapes of minimal energy. This means that the area of a soap film is the smallest area that you can find. He was right.
15. Get your favourite wire frame and blow a soap film in it.

(a) Can you obtain a different soap film by blowing, pricking or moving it?

(b) You may find that other shapes are possible, but each of them is a surface of minimal area! How can you explain this?

Your teacher will conclude this lesson with a short summary about the three laws of Plateau.

3.3 Worksheets

Below the two worksheets are given in separate pages. You can make hard copies of them and hand them out to your pupils.
Worksheet 2

A

B

C

D
References


